THE THEORY OF ATMOSPHERIC TURBULENCE—AN HISTORICAL RESUMÉ AND AN OUTLOOK

By CARL-GUSTAF ROSSBY

[Weather Bureau, Washington, March 2, 1927]

Everyone knows that the air is only seldom in a state of uniform motion. The smoke from a chimney and the dust whirls on the ground show that the wind practically always is varying rapidly in velocity and direction. Even on quiet summer days with apparently no wind at all, small irregular gusts set the leaves trembling. Examination of a few anemometer records will confirm these simple observations and at the same time show that the degree of irregularity varies from time to time.

We attribute these variations of wind velocity and wind direction to the occurrence of numerous eddies in the atmosphere, saying that the air generally is in a state of turbulence or that the atmospheric currents are turbulent. The study of turbulence has in the last two decades gradually grown to be a distinct branch of meteorology and the results have found widespread application in different fields.

Our present knowledge of atmospheric turbulence is only to a limited extent a fruit of direct studies of anemometer records; far more is it a result of theoretical discussions of the internal friction of the air, supplemented by analyses of curves for the vertical distribution of wind velocity. We shall here try to outline the development of this branch and also to indicate certain directions in which further work seems desirable.

In his first paper on "Atmosphärische Bewegungen" Helmholtz (*1*) showed that if we want to account for the rapid dissipation of kinetic energy in the atmosphere (demonstrated by the short life of storms), the molecular viscosity of the air as determined by laboratory experiments is entirely too small. As an illustration, Helmholtz took the case of a simple laminar motion of the atmosphere under the sole influence of viscosity. He determined the vertical velocity distribution from the ordinary hydrodynamical equations in the form Euler and Navier had given them and then computed the time in which the velocity at all levels would decrease by one half.[1] Using the laboratory value for the molecular viscosity of air, he obtained the amazing result that it would require 42,747 years. To explain the immensely more rapid dissipation actually observed, Helmholtz started from the conception of atmospheric surfaces of discontinuity, in other words surfaces where density and temperature suddenly change. Having derived the equilibrium conditions for these surfaces he showed that they can exist only for a short time, because they are unstable, and therefore under the influence of small disturbing forces roll up in vortices. "In these vortices the originally separated air masses are folded around each other in more and more numerous and therefore thinner and thinner layers, and in this way, through the tremendously magnified contact surface, a rapid equalization of temperature and velocity is possible."

After Helmholtz had shown the inadequacy of the molecular viscosity term in the classical Euler-Navier equations when applied to large atmospheric movements, Guldberg and Mohn (*2*) attempted to find other equations containing frictional terms which would better account for the dissipation of kinetic energy within the atmosphere.

According to these authors the friction is simply proportional and opposite to the wind velocity. Their equations, which describe comparatively well the movements of the surface layers, in which the chief frictional influence is the resistance from the ground, were improved by Sandström (*3*). He showed that the frictional force acting on the surface layer is not exactly opposite to the wind direction, but is deviated to the right (in the northern hemisphere) and explained this deviation as the effect of the frictional drag exerted by the upper layers. From measurements on synoptic maps he found the average value of the angle between the total frictional force on the surface layer and the reversed wind direction at the surface to be about 38°.

However, these semiempirical equations served only the purpose of describing the movements of the surface layer and could not contribute to a deeper understanding of atmospheric movements, and especially of the nature of the frictional interaction between different horizontal layers. The first fundamental step toward the solution of the latter problem was taken by Åkerblom (*4*). His starting point was the assumption that the Euler-Navier equations in principle describe atmospheric movements correctly if we substitute for the coefficient of molecular viscosity another, suitable coefficient characterizing the apparent or *virtual* atmospheric viscosity. Under this assumption Åkerblom integrated the dynamical equations for the simple case of rectilinear motion with straight parallel isobars and obtained a certain curve for the vertical variation of wind velocity and wind direction.[2] Comparing the theoretical results with observational data from the Eiffel Tower, he could determine the coefficient of the virtual viscosity. This turned out to be many thousand times larger than the molecular viscosity. Thus the molecular viscosity of air at 0° C. is equal to 0.000170 $\frac{\text{gram}}{\text{cm. sec.}}$, whereas the values found by Åkerblom vary between 87 $\frac{\text{gram}}{\text{cm. sec.}}$ in winter and 113 $\frac{\text{gram}}{\text{cm. sec.}}$ in summer. Using these new values Åkerblom obtained a close agreement between the theoretical and the observed curves. Thus he had proved that if a suitable virtual viscosity coefficient is used, the Euler-Navier equations can be applied also to the study of large atmospheric movements. As seen from the numerical values given above, Åkerblom also found that the virtual friction has a marked seasonal variation.

Akerblom's ideas were further developed by Hesselberg and Sverdrup (*6*). They had at their disposal the abundant aerological material from the observatory at Lindenberg and could determine the value of the virtual viscosity for different layers. They found that the coefficient increases from a very small value close to the surface up to

[1] In the equation for the movement along the x-axis (assumed horizontal) the viscosity term has the form

$$\mu\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}+\frac{\delta^2 u}{\delta z^2}\right),$$

where μ is the coefficient of molecular viscosity, and u is the velocity component along the x-axis. In the atmosphere, where the horizontal variations of velocity are small compared with the vertical, the first two terms can generally be neglected (the z-axis is assumed vertical). Thus the frictional term reduces to

$$\mu\frac{\delta^2 u}{\delta z^2}$$

[2] Åkerblom was led to his investigation through a paper by V. W. Ekman (*5*), who had already solved theoretically a corresponding problem for the sea, i. e. the determination of the vertical distribution of velocity in a drift current.

a many times greater value at about 300 meters; from here on it remains comparatively constant with height.[3]

So far the values of the virtual coefficient of viscosity had all been obtained from the ordinary hydrodynamical equations. These equations simply tell what the velocity distribution will be if the virtual friction has a certain value. They, therefore, allow us to draw certain conclusions from an observed velocity distribution as to the value of the virtual friction. However, they do not tell us the physical factors that produce the virtual friction. Now Åkerblom's, as well as Hesselberg's and Sverdrup's work had shown that the virtual friction varies in space and time. The question of the causes of this virtual friction and its variations therefore naturally arose. Investigations with this definite program were not at once taken up. However, some very important studies were made which in a way prepared for the solution of the new fundamental problem.

In 1915 Taylor published a paper (*9*), which for the development which was to follow became of highest importance. Hesselberg and Sverdrup recognized that the virtual friction was due to turbulence, in other words to the numerous small eddies superposed on the larger atmospheric movements, but they did not show how the turbulence could produce such an effect. This was the task to which Taylor set himself.

Taylor started from the fact that there is constantly a more or less intensive exchange of mass between the different (horizontal) air layers. Small eddies are constantly leaving their mother layer and traveling upward or downward until they gradually are assimilated by the surroundings. This exchange of mass is accompanied by an exchange of the properties characteristic of the different air layers, for instance, their water vapor content, heat and horizontal momentum. As a result there will be a transport of, for instance, water vapor upward if this element decreases with height, and downward if it increases with height. The magnitude of the upgrade current of water vapor passing through a horizontal unit surface per second is equal to

$$-c\frac{\delta m}{\delta z},$$

where c is a coefficient, called by Taylor "eddy conductivity," and m is the water vapor content in grams per gram of moist air. When the same reasoning is applied to the temperature (which was the element first treated by Taylor) it must be borne in mind that this quantity is not an invariant, characteristic of the layer, as is for instance the case with the amount of water vapor per gram of moist air (as long as no condensation takes place). On the contrary, when an eddy leaves the mother layer on its upward or downward route it is expanded or compressed adiabatically, thus all the time changing its temperature. We know, however, that the *potential temperature* (θ) remains constant during all adiabatic processes. This quantity, therefore, is an invariant, characteristic of the mother layer of the eddy, and the reasoning above may therefore be applied also to the discussion of eddy transport of potential temperature. The transport upward of potential temperature is, per unit cross section, equal to

$$-c\frac{\delta \theta}{\delta z},$$

where the coefficient c probably has the same value as in the expression for the transport of water vapor. The important result of this application is, that as long as the potential temperature increases with height, as it practically always does, the turbulence carries potential temperature and therefore heat *downward*. Only in the case of unstable stratification (upward decreasing potential temperature) do the eddies carry heat *upward*.

Applying the same method to the horizontal momentum, Taylor found for the gain of momentum in a certain layer an expression of the same form as the principal part of the frictional term in the Euler-Navier equations. Thus it was proved, that the turbulent exchange of mass between different layers is responsible for the virtual friction in the atmosphere and a new independent proof had been obtained for the validity of the Euler-Navier equations, provided only a proper viscosity coefficient is used. Taylor introduced instead of the term virtual friction the name "eddy viscosity," which now is generally accepted among English writers.

For the vertical transport of horizontal momentum through unit surface he obtained the expressions

$$-\mu\frac{\delta u}{\delta z}, \quad -\mu\frac{\delta v}{\delta z}.$$

Here u and v are the two horizontal velocity components and μ the coefficient of eddy viscosity. Taylor suggested for theoretical reasons that eddy viscosity and eddy conductivity are equal. His own as well as later measurements seem to confirm this view.

A little later V. Schmidt (*10*) published a study of a very similar nature and reached the same conclusions as Taylor. Schmidt has worked intensively on the application of the equations for eddy convection of heat and water vapor to various questions. He discussed, for instance, the diurnal variation of temperature in the free atmosphere, the influence of large cities on climate, etc. He, more than anyone else, has shown through his investigations the wide applicability of these mathematical methods to the most varying problems.

With only slight alterations the methods of Schmidt and Taylor may be applied to the study of the horizontal transport of heat and water vapor from the equator toward the poles. In this latter case cyclones and anticyclones are regarded as eddies superposed upon a general West-East circulation. While on account of the entirely different order of magnitude of cyclones and anticyclones as compared with the eddies of the free atmosphere, their coefficient of eddy conductivity has values of a different order of magnitude, nevertheless the principles of the treatment remain the same. This horizontal convection problem was taken up by Defant (*11*), Exner (*12*), Ångström (*13*). Schmidt introduced the term "Austausch" (exchange), which in the main

[3] The latter conclusion, namely, that the coefficient of virtual friction from a few hundred meters above surface remains constant with height, seems not justifiable. Using the notations introduced in footnote 1 we have for the x-component of the frictional stress per horizontal unit surface the expression $\mu\frac{\delta u}{\delta z}$. Now, since μ is variable, it is easily proved that the resultant frictional force ~~per~~ unit layer will be equal to $\frac{\delta}{\delta z}\left(\mu\frac{\delta u}{\delta z}\right)$. In computing their μ-values Hesselberg and Sverdrup replaced this expression by $\mu\frac{\delta^2 u}{\delta z^2}$, thus neglecting the term $\frac{\delta \mu}{\delta z}\cdot\frac{\delta u}{\delta z}$ which is of the same order of magnitude as $\mu\frac{\delta^2 u}{\delta z^2}$. That this approximation highly affects the results of the computation has been shown in another place (*7*). There is, moreover, a principal difference between a frictional term of the type $\mu\frac{\delta^2 u}{\delta z^2}$ and one of the type $\frac{\delta}{\delta z}\left(\mu\frac{\delta u}{\delta z}\right)$. The former will, since μ is essentially positive, always have the same sign as $\frac{\delta^2 u}{\delta z^2}$, whether μ *is variable or not, and will therefore always tend* to annihilate existing differences of velocity. The latter expression may, however, on account of the term $\frac{\delta \mu}{\delta z}\frac{\delta u}{\delta z}$ have the opposite sign to $\frac{\delta^2 u}{\delta z^2}$. In this case differences of velocity already existing may under favorable conditions increase and finally develop into real discontinuities (*8*).

coincides with Taylor's "eddy conductivity" and which is generally used by German writers.

Taylor's and Schmidt's work immediately stimulated a very active study of the eddy activity in the atmosphere. L. F. Richardson, one of the first to take up the problem after them, computed by means of the most varied methods a number of values of the eddy conductivity and eddy viscosity. A very complete table of his own values as well as those obtained by other authors is published in his book "Weather Prediction by Numeral Process" (*14*). These values vary between such wide limits as 0.001 and 1030 $\frac{\text{grams}}{\text{cm.} \times \text{sec.}}$. Thus it is evident that a value of the eddy conductivity without indication of the circumstances under which it was obtained is useless. By "circumstances" in this case we mean the conditions which determine the state of turbulence; thus, we are again led back to our fundamental question. However, Richardson's tables seem to indicate as a general conclusion that the eddy conductivity is very small close to the surface, then increases to a maximum within the first kilometer, and finally at the higher levels again decreases (compare footnote 3).

His data also show another peculiarity, which may throw a certain light on the arrangement of the atmospheric eddies. If we place at a certain level a coordinate system with the x-axis in the direction of the wind and the z-axis along the vertical, then we have at this level,

$u \neq 0$, $v=0$ (u, v, w denote the velocity components).

Since, however, wind direction and wind velocity vary with height, we have

$$\frac{\delta u}{\delta z} \neq 0, \ \frac{\delta v}{\delta z} \quad 0.$$

The eddy stress per cm.[2] at the level $z=0$ has the components

$$-\mu\frac{\delta u}{\delta z}$$

and

$$-\mu\frac{\delta v}{\delta z}$$

Richardson found that μ does not have the same value in both these expressions, but is about seven times larger in the latter case. Thus there is a small resistance against a variation of the velocity with height, but a very strong resistance against a change in wind direction. We can understand this if we imagine that the eddies are arranged in long rolls with horizontal axes, everywhere orthogonal to the wind direction. Just as only a small impulse is needed to push a wagon forward or backward, while very great force must be applied to push it sideward, in the same way the upper layer rolls easily over the lower in the direction of the wind, but there is a great resistance when it tries to move in another direction. It is obvious that this lack of isotropy in the arrangement of the eddies, which has not yet been systematically investigated, besides being important for the dynamics of the atmosphere, should also be taken into consideration in computing the resistance against bodies moving through the air. An airship moving in the wind direction and one moving at right angles to it, should not, other things being equal, experience the same resistance.

Richardson was the first to take up the general problem of determining the physical factors which create turbulence. In his paper "The Supply of Energy to and from Atmospheric Eddies" (*15*), he sets himself the task of finding a criterion for the conditions under which the kinetic energy of eddies, in brief the turbulent energy, will decrease or increase. This idea was in itself not new; it had been used by Osborn Reynolds, who had derived a Criterion of Turbulence for an incompressible liquid. In the same way as in a small scale hydrodynamical experiment molecular viscosity is transforming the kinetic energy of the visible movement into invisible molecular kinetic energy, i. e. heat, so eddy viscosity transforms the kinetic energy of the regular movement into kinetic energy of turbulence. Reynold's theorem says that the increase of the total turbulent energy within a closed system must be equal to the gain of turbulent energy from the mean motion (through eddy viscosity) minus the loss of turbulent energy (through molecular viscosity). The latter transforms turbulent energy into heat.

In the application of this theorem to the atmosphere Richardson had to generalize it considerably. The atmosphere is not incompressible, but is an ideal gas under the influence of an external field of force, gravity. Thus he had to take two new energy forms into consideration, potential and internal. These two forms of energy are however closely connected. Dines (and before him Margules) had shown that the total change of potential energy within a vertical air column, limited by fixed walls, always stands in a constant proportion to the change of internal energy.[4] Now it is easy to see that in general the activity of the eddies increases the potential and therefore also the internal energy of the atmosphere. If the lapse rate is stable, then a small eddy, lifted from mother layer, will arrive in the new layer colder than the surroundings and therefore try to sink. Energy is therefore needed to produce the lifting and it is drawn from the kinetic energy of the eddy which thus is transformed into potential energy. Similarly, an eddy moving downward from its mother layer will everywhere arrive warmer than the surroundings and therefore try to rise. Work is again needed and is taken from the kinetic energy of the eddy. Richardson showed that the loss of turbulent energy through this work against the generally stable vertical stratification, that is, against gravity, is much more important than the loss through the action of molecular viscosity, which all the time, but very slowly, is transforming turbulent kinetic energy into heat. It is, therefore, in most cases permissible to neglect entirely the molecular viscosity.

Now the production per unit volume of turbulent energy from the kinetic energy of the mean motion is equal to

$$\mu\left[\left(\frac{\delta u}{\delta z}\right)^2+\left(\frac{\delta v}{\delta z}\right)^2\right]$$

For the loss of turbulent energy through work against the stratification, Richardson finds the expression [5]

$$cg\,\frac{1}{\theta}\,\frac{\delta\theta}{\delta z} \quad (g = \text{acceleration of gravity})$$

[4] If the potential energy is denoted by P, the internal by I, and a variation is indicated by δ, then

$$\delta P = \frac{R}{m\,c_v}\delta I$$

Here R is the absolute gas constant, m the molecular weight of air and c_v the specific heat at constant volume.

[5] The expression originally given by Richardson contains the vertical lapse rate of specific entropy; in case of dry or clear moist air it can, however, with only slight error be transformed into the expression given above. This latter expression shows, that in case of upward decreasing potential temperature, potential and internal energy are transformed into eddy energy.

Thus, in order that the turbulent energy may decrease at a certain point, we must have

$$\mu\left[\left(\frac{\delta u}{\delta z}\right)^2+\left(\frac{\delta v}{\delta z}\right)^2\right] < cg\,\frac{1}{\theta}\,\frac{\delta\theta}{\delta z}$$

If we assume μ and c, eddy viscosity and eddy conductivity, to be equal, then the above expression can be still more simplified. Richardson's theorem may be formulated in words:

To every value of the vertical lapse rate of temperature there corresponds a critical value for the increase of wind with height. If the increase actually observed is less than the critical value, then the turbulence (more definitely the eddy energy) has a tendency to decrease. If the vertical increase of wind velocity exceeds the critical value, then the eddy energy has a tendency to increase.

For an average clear day the critical value (in the troposphere) is equal to 1 m/sec. per 100 m.

Richardson's criterion closely resembles the definition of the critical Reynold's number. In investigating the flow of a liquid, for instance water, through a narrow pipe of given diameter D, we mean by Reynold's number the quantity

$$\frac{\rho UD}{\mu}.$$

ρ is the density, μ the molecular viscosity and U the mean velocity through a cross-section of the pipe. As long as U remains small, the flow will be laminar, but when U passes a certain critical value, the laminar flow brakes up into eddies and becomes turbulent.

Richardson in the paper already cited (*15*) also laid down the fundamentals of what he called "Eddy-Thermodynamics." He proved that if a volume, containing numerous eddies, is expanded or compressed adiabatically the total turbulent energy (and therefore also the eddy energy per unit mass E) will vary with the density ρ according to the law[6]

$$E = \text{const.}\ \rho^{\frac{2}{3}}$$

This relation has the same form as the relation between temperature and density in a monatomic gas which is expanded adiabatically. It has not yet found any practical application, but Richardson points out that it may have some bearing upon the state of turbulence in an air mass flowing up or down a hillside or entering into a huge convection cloud.

Finally, Richardson pointed out, that there is in the atmosphere not only a production and consumption of eddies but also a diffusion of them from regions where they are crowded to regions where they are rare. As an example he gave the experience of an aviator flying at about 700 m. over the Nile. Early in the morning the air was calm and the river smooth as a mirror, but when the sun began to heat the ground, small ripples formed on the surface of the water. About half an hour later the aviator could feel the first gusts from the rising turbulence. In the air, eddies of all sizes are present. The larger ones, traveling from one layer to another, will carry numerous smaller eddies with them. The diffusion of these small eddies, of which there may be a great number in one large eddy, probably follows a law similar to that given for the turbulent transport of potential temperature. Whether the diffusion of large eddies obeys the same law seems impossible to prove in any other way than by the success or failure of a theory based on that assumption.

Now let us sum up what had been achieved by Richardson and his predecessors and what remained to be done. The gains were:

1. An expression had been derived for the production of eddy energy from the kinetic energy of the mean motion.
2. An expression had been derived for the loss of eddy energy through work against a stable stratification, and for the gain of eddy energy in case of an unstable stratification.
3. A criterion had been found for the conditions under which an air current will remain laminar or become turbulent.
4. An equation of continuity had been derived, expressing the increase of eddy energy in a closed system as the difference between production and consumption of eddies within the system.

The shortcomings of the theory as an aid to the solution of the general problem are

1. The state of turbulence is characterized by three different quantities,

   eddy viscosity μ
   eddy conductivity c
   eddy energy E

2. Between these three characteristics there is as yet only one relation, the energy equation. Thus two of the above characteristics of the turbulence remain undetermined.
3. The energy equation refers to a closed system and can therefore not be used for computation of the state of turbulence at individual points.

The reason why Richardson's equation can not be applied to any small element of volume is obvious. Such an element can not be regarded as a closed system. There is diffusion of eddy energy through the boundaries of the elements; therefore new assumptions concerning this diffusion have to be made and perhaps new characteristics of the turbulence introduced.

In two contributions to this journal (*16, 17*) I have tried to modify the theory in a way to take care of the difficulties mentioned above. Naturally, a number of assumptions were necessary, justification of which must be sought in the results of the theory. The fundamental assumptions are:

1. Eddy energy per unit mass (E), which quantity for brevity's sake may be called specific eddy energy, is diffused upward per cm² and sec. at the rate of

$$-c_E\,\frac{\delta E}{\delta z} \qquad (c_E \text{ is a coefficient})$$

2. The characteristics of the turbulence thus introduced,

   eddy viscosity μ
   eddy conductivity c
   diffusion coefficient c_E
   specific eddy energy E

are reduced to one, E, through the assumption that μ, c, and c_E are all proportional to E.

[6] This formula suggests the introduction of a quantity $\Xi = E.\ \rho^{-\frac{2}{3}}$, which remains constant during adiabatic expansion or compression of the air. Richardson gave Ξ the name "potential eddy-heat per unit mass."

The expression for the diffusion current of eddy energy is formed in a way analogous to the corresponding expressions for the diffusion of other meteorological elements. Since, however, not the specific eddy energy but the potential eddy energy (see footnote 6) remains constant during the transport of an air mass from one layer to another, it would perhaps have been more correct to introduce the latter quantity in the expression for the diffusion current.

The assumed proportionality between μ, c, c_E and E is probably a good approximation for regions of the free atmosphere where the linear dimensions of the eddies differ but little from point to point. Approaching the ground, however, we find that the eddies rapidly decrease in size. The assumption is therefore probably not fulfilled in this region.

It is easily seen that if the energy equation is applied to a small volume element and the diffusion through the boundaries taken into account, this equation will become a differential one. If, furthermore, we introduce E everywhere instead of μ, c, and c_E, we obtain a differential equation which permits us to compute the distribution of the specific eddy energy at any time, if this distribution is known at a certain moment. The coefficients of the new differential equation contain the quantities

$$\left(\frac{\delta u}{\delta z}\right)^2+\left(\frac{\delta v}{\delta z}\right) \text{and} \frac{1}{\theta}\frac{\delta\theta}{\delta z}$$

in other words, the increase of wind velocity and potential temperature with height. Thus the solution of our differential equation gives us the specific eddy energy as a function of the vertical distribution of temperature and wind.

As a test of this theory the equation for E was integrated for a number of simple cases. Thus the diffusion under different conditions of an originally limited supply of eddy energy was discussed and qualitatively satisfactory results were obtained. Assuming a simple vertical distribution of temperature and wind velocity reasonably corresponding to normal atmospheric conditions, a curve was derived for the vertical distribution of the specific eddy energy and thus, by virtue of our second assumption, also for the eddy viscosity. This curve agreed well with measurements of the vertical distribution of eddy viscosity (*18*).

If, in the case of no mean motion, the equation for E is combined with the equation for eddy convection of heat, a system is obtained, containing two variables, E, the specific eddy energy, and θ, the potential temperature. This system furnishes us with the general solution of the problem of thermal convection. The solution may, however, not be applied to the discussion of, for instance, the growth of an individual cumulus cloud. The whole theory is statistical, i. e., it deals with the behavior of a number of eddies, not with the growth and decay of individual eddies.

For a real test of the theory it would be highly desirable to get some values of the specific eddy energy at different levels. Such values could probably be computed from records obtained by anemometers or other instruments registering gustiness. It would be especially desirable to have such records from some distance above the ground, since the dimensions of the eddies in the surface layers are small compared with those of the free atmosphere, More observations of the rate of diffusion of turbulence. of the same type as the example quoted from Richardson, would also aid the development of the theory.

Simultaneous with these observations of specific eddy energy and the rate of diffusion, we should measure the vertical distribution of wind and temperature as well as eddy viscosity. I hope later to discuss more fully measurements referring to turbulence in general.

LITERATURE CITED

(*1*) HELMHOLTZ, H. VON
1895. WISSENSCHAFTLICHE ABHANDLUNGEN III. p. 289, Leipzig.

(*2*) GULDBERG ET MOHN.
1876–1880. ÉTUDES SUR LES MOUVEMENTS DE L'ATMOSPHÈRE. Christiana.

(*3*) SANDSTRÖM, J. W.
1913. EINE METEOROLOGISCHE FORSCHUNGSREISE IN DEM SCHWEDISCHEN HOCHGEBIRGE. K. Sv. Vet. Ak. Handl. Bd. 50, No. 9, Stockholm.

(*4*) ÅKERBLOM, F.
1908. RECHERCHES SUR LES COURANTS LES PLUS BAS DE L'ATMOSPHÈRE AU DESSUS DE PARIS. Nova Acta Reg. Soc. Uppsala.

(*5*) EKMAN, V. W.
1905. ON THE INFLUENCE OF THE EARTH'S *ROTATION ON* OCEAN CURRENTS. Arkiv för Mat. Astr. och Fysik, Bd. 2, No. 11, Stockholm.

(*6*) HESSELBERG, TH. UND SVERDRUP, H. U.
1915. DIE REIBUNG IN DER ATMOSPHÄRE. VERÖFF. DES GEOPHYSIK. INST. DER UNIVERSITÄT LEIPZIG.

(*7*) ROSSBY, C.-G.
1925. METEOROLOGISKA RESULTAT AF EN SOMMARSEGLATS RUNT DE BRITTISKA ÖARNA. Statens Met.—Hydr. Anstalt, Medd. Bd. 3, No. 1. Stockholm.

(*8*) ROSSBY, C.-G.
1924. ON THE ORIGIN OF TRAVELLING DISCONTINUITIES IN THE ATMOSPHERE. Geogr. Annaler, vol. 6, p. 180. Stockholm.

(*9*) TAYLOR, G. I.
1915. EDDY MOTION IN THE ATMOSPHERE. *Phil. Trans.* vol. 215, I, London.

(*10*) SCHMIDT, W.
1917. DER MASSENAUSTAUSCH BEI DER UNGEORDNETEN STRÖMING IN FREIER LUFT UND SEINE FOLGEN. Wiener Sitzber, II a, 126.

(*11*) DEFANT, A.
1921. DIE ZIRKULATION IN DER ATMOSPHÄRE IN DEN GEMÄSSIGTEN BREITEN DER ERDE. Geografiska Annaler, vol. 3, p. 209. Stockholm.

(*12*) EXNER, F.
1925. DYNAMISCHE METEOROLOGIE. Zweite Auflage, p. 239, Vienna.

(*13*) ÅNGSTRÖM, A.
1926. EVAPORATION AND PRECIPITATION AT VARIOUS LATITUDES AND THE HORIZONTAL EDDY CONDUCTIVITY OF THE ATMOSPHERE. Arkiv för Mat. Astr. och Fysik, Bd. 19, No. 20.

(*14*) RICHARDSON, L. F.
1922. WEATHER PREDICTION BY NUMERICAL PROCESS. Cambridge.

(*15*) RICHARDSON, L. F.
1920. THE SUPPLY OF ENERGY TO AND FROM ATMOSPHERIC EDDIES. Proceedings Royal Soc. A, vol. 97, London.

(*16*) ROSSBY, C.-G.
1926. THE VERTICAL DISTRIBUTION OF ATMOSPHERIC EDDY ENERGY. Mo. Wea. Rev. *54*, No. 8, Washington.

(*17*) ROSSBY, C.-G.
1927. CONVECTION IN THE FREE ATMOSPHERE AND OVER A HEATED SURFACE. Mo. Wea. Rev. *55*, No. 1, Washington.

(*18*) SOLBERG, H.
1923. SUR LE FROTTEMENT DANS LES COUCHES BASSES DE L'ATMOSPHÈRE. Förh. Skand. Naturforskarmötet i Göteborg, p. 95.